



Figure 3. Schematic of a cooling channel with relevant wall temperatures and heat transfer rates (the symbol Q indicates the heat transfer per unit length of the chamber). Note that the dimensions refer to a section that is orthogonal to the coolant flow.

According to the proposed modeling, the heat transfer rates per unit length of the chamber pertaining to the outer wall are:

$$\begin{cases} Q_{r,t} = w \frac{k}{d_r} (T_{r,t} - T_{r,o}) \\ Q_{rc} = d \frac{k}{l_o} (T_{r,o} - T_{c,o}) \\ Q_{c,t} = b \frac{k}{d_r} (T_{c,o} - T_{c,t}) = bh_c (T_{c,t} - T_c) \end{cases} \quad (3)$$

Considering that the balance of the heat transfer rates is $Q_{r,t} = 2Q_{rc} = Q_{c,t}$, the heat flux at the tip of the rib, $q_{r,t} = Q_{r,t}/w$, can be computed, if the coolant and the rib tip temperatures are known, as:

$$q_{r,t} = H(T_{r,t} - T_c) \quad \text{where} \quad H = \left(\frac{d_r}{k} + \frac{wl_o}{2dk} + \frac{wd_r}{bk} + \frac{w}{bh_c} \right)^{-1} \quad (4)$$

The outer wall temperatures can be computed with the following sequence:

$$\begin{cases} T_{r,o} = T_{r,t} - \frac{Hd_r}{k} (T_{r,t} - T_c) \\ T_{c,o} = T_{r,o} - \frac{Hwl_o}{4kd} (T_{r,t} - T_c) \rightarrow 2kd \text{ (NON } 4kd) \\ T_{c,t} = T_{c,o} - \frac{Hwd_r}{kb} (T_{r,t} - T_c) \end{cases} \quad (5)$$

Using Equations (1), (2) and (4), the wall temperature and the heat flux at the base of the rib are:

$$T_{r,b} = T_c + \frac{T_{r,t} - T_c}{\beta} \quad \text{and} \quad q_{r,b} = km(T_{r,b} - T_c) \frac{\cosh(mh) - \beta}{\sinh(mh)} \quad (6)$$